Handling Exceptions in Nonmonotonic Reasoning

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Abstract We introduce some differences in the style defeasible information is represented and inferences are made in nonmonotonic reasoning. These, at first sight harmless, changes, in fact, helped us to discover a very important principle guiding how inferences should be drawn in nonmonotonic reasoning, we name it the *exception-first principle* or *EFP*. `DLEF` is our own variant for default logic complying with EFP. We also show alternative definitions for Reiter's default logics and its justified and constrained variants within our framework. `DLEF` does not produce anomalous extensions where the other default logics do. Restricted to the language of general logic programs, `DLEF`, Reiter's default logic and answer set programming all coincide. This explains why ASP is appointed as a solution to the anomalous extensions problem, ASP complies with EFP.

1 Introduction

We introduce some differences in the style defeasible information is represented and inferences are made in nonmonotonic reasoning, hereafter NMR, for short. These at first sight harmless changes, in fact, helped us to discover a very important principle guiding how inferences are (or should be) drawn in nonmonotonic reasoning, we name it the *exceptions-first principle* or EFP.

As it is usual in nonmonotonic reasoning in artificial intelligence (AI), we assume a knowledge basis comprehending certain and defeasible information, by defeasible information we mean information that may be withdrawn in the light of new information. However, we see that what is normally called a *default*[1] in NMR literature, not as an inference rule, but as a *proposition subject to exceptions*. We regard an exception, then, as a precondition to the use of a proposition as a valid premise in a reasoning course. It is expected in NMR that defeasible propositions clash with each other: they might be jointly inconsistent or even one proposition may prove an exception to the other. The exceptions-first principle simply states that the exception - being a precondition to the proposition - must be verified prior to the use of the respective proposition. Thus, in our viewpoint it is inconceivable that a proposition in a course of reasoning

[1] The default logic was proposed by Raymond Reiter in [20].

may interfere with the derivation of its own exception, either supporting or ruling it out. Hence, in case of conflict between a proposition and its exception, it is the exception, not the proposition, that is applied.

This might seem natural but none of the seminal nonmonotonic logics[2] comply with this principle. This issue is in the core of the so-called *anomalous extensions problem*, which challenged the nonmonotonic logics to correctly represent the frame problem in the late 80's [8,2].

Accordingly to this view, we regard a defeasible proposition as a pair of formulae (P, Q), where P is a proposition and Q is an exception to it. To represent a defeasible proposition we use the notation $P ─(Q$, read as "generally P, unless Q is the case", and call it a generalization. P is called the conjecture and Q the exception or restriction. The reader familiar with Reiter's default logic [20] may consider generalization $P ─(Q$ as corresponding to the prerequisite-free multiple semi-normal default $\frac{:P, \neg Q}{P}$. For instance, the defeasible proposition "normally, birds fly unless they are penguins", which in default logic is represented as the default $\frac{bird : fly \wedge \neg penguin}{fly}$, in our approach it is represented by the generalization $bird \rightarrow fly ─(penguin$, which corresponds to the prerequisite-free default $\frac{:bird \rightarrow fly, \neg penguin}{bird \rightarrow fly}$, as we shall show in the sequel.

Now, given a knowledge basis comprehending certain and defeasible propositions - we shall call it *a defeasible axiomatic basis* - what are the inferences supported by this basis? The intuitive normative is "to extract as much information as possible without getting into inconsistency or violating exceptions". Different approaches to how to do that give rise to different consistency based formalisms such as default logic and its variants. A key issue is how to accommodate conflicting evidence among the defeasible propositions. One and the same formula may appear as a conjecture in a generalization and as an exception in another. The interplay between defeasible propositions and exceptions plays a central role in NMR. The EFP regulates how this interaction must be done, lest unsupported inferences are drawn.

Thus, in our approach sets of generalizations are picked up as *candidates* to generate the theorems of a defeasible axiomatic basis[3]. Candidates actually generate the theorems of a defeasible axiomatic basis when they satisfy some properties that specify what we call an *expansion*.

These properties defining expansion specify which information is inferred from a defeasible axiomatic basis. Different properties give rise to different nonmonotonic logics as we shall show in the sequel for Reiter's default logic and two of its variants: Lukaszewicz's justified [11] and Delgrande's constrained default logics [3].

The normative of extracting as much defeasible information as possible preserving consistency and observing exceptions is pursued for all nonmonotonic

[2] By seminal nonmonotonic logics we refer to Reiter's Default Logic, McCarhty's Circumscription and Doyle-McDermott's Modal Nonmonotonic Logic. These logics were published in the special issue of the Journal of Artificial Intelligence which launched the NMR area in 1980 (vol. 13, numbers 1 and 2).

[3] Candidates correspond to set of defaults in Reiter's default logic.

logics studied here. A property of soundness (the observance of exceptions and consistency) and one of maximality (including as much defeasible propositions as possible while observing soundness) is common to all of these logics. Yet, each logic presents a different specification for them. Compliance with the EFP is exclusive to the Defeasible Logic with Exceptions-First (`DLEF`, for short), our own logic presented in this paper. That's why, we argue, all of these default logics, but `DLEF`, generate unsupported inferences. The handling of exceptions is what differentiates these logics since all of them coincide for normal theories, the ones where exceptions for defeasible propositions are not represented.

It has been argued that the answer set semantics (ASP, for short) in logic programming solves the problem of anomalous extensions, which are derived in default logic [21]. We show, in Section 6 that ASP is, in fact, the restriction of Reiter's default logic for the language of logic programs. Only because logic programs are less expressive than default logic, the anomalous extensions do not manifest in ASP. Moreover, restricted to stratified logic programs, `DLEF` also coincides with ASP. Compliance with EFP is the reason why there are no anomalous extensions in ASP.

This paper is organized as follows. In Sect. 2 technical definitions concerning the logical framework are given. Properties that characterize expansions are defined in Sect. 3. In Sect. 4 we present our own logic `DLEF`. Section 5 presents different versions of expansions showing how they correspond to default logic and its variants. In section 6, we relate our work with ASP. In section 7 we outline future work to be done.

2 Technical Preliminaries

We adopt a propositional language L[4] augmented by a new operator "–(" to represent defeasible propositions in the form of generalizations, as defined below.

Definition 2.1. *A generalization in L is an expression of the form P–(Q such that P, Q are formulas of L[5].*

A generalization g of the form P–(Q is read as "generally P, unless Q is the case". 'P' is called the conjecture of g, and 'Q' the exception or restriction of g. We write P –(, when the restriction is a contradiction.

Definition 2.2. *Let g be a generalization of the form P–(Q then it is specified:*

1. *Conj(g) $\equiv P$.*
2. *Rest(g) $\equiv Q$.*

A defeasible axiomatic basis is a pair of sets of formulas and generalizations representing certain and defeasible information, respectively.

[4] For simplicity's sake, we adopt a propositional language, although the formalism is readily lifted to the predicate case.

[5] So P and Q do not contain the connective –(.

Definition 2.3. *A defeasible axiomatic basis τ in L is a pair $\tau = (W, G)$, whereon W is a collection of formulas of L, and G is a finite*[6] *collection of generalizations.*

The reader familiar with Reiter's default logic [20] may recognize a generalization as a prerequisite-free semi-normal multiple default, this is exactly our intention as shown by the translation scheme from defeasible axiomatic basis into default theories below.

Definition 2.4. *Let $\tau = (W, G)$ be a defeasible axiomatic basis, $\Delta = (W, D)$ is the corresponding default theory where:*[7]

$$D = \left\{ \frac{: P, \neg Q}{P} \in D \mid P -(\ Q \in G \right\}$$

In this manner, our defeasible axiomatic basis have all advantages that prerequisite-free semi-normal default theories have. The main feature is that semi-normal defaults possess the same expression power of general defaults [9]. Besides, prerequisite-free defaults enjoy the same properties as classical formulae. As, for example, contraposition, reasoning by cases etc. In designing `DLEF`, we adopt the methodological principal to keep it as close to classical logic as possible. We consider this linear notation more convenient and elegant.

Hereafter, we assume that all definitions refer to a defeasible axiomatic basis $\tau = (W, G)$, unless otherwise stated.

Our approach for determining the theorems supported by a defeasible axiomatic basis is to determine maximal collections of compatible generalizations, so collections of generalizations play an important role in our formalism. We call then as *candidates in τ*. The motivation for this name is that collections of generalizations are candidates to be expansions in τ.

Definition 2.5. *A candidate γ in a defeasible axiomatic basis $\tau = (W, G)$ is a collection of generalizations in τ, that is, $\gamma \subseteq G$.*

Example 2.1. Let $\tau = (\emptyset, \{P -(\ Q, Q -(\ R, \neg P -(\ \})$ Some candidates to be expansions are: $\gamma_1 = \{Q -(\ R, \neg P -(\ \}$; $\gamma_2 = \{P -(\ Q, Q -(\ R\}$; $\gamma_3 = \{P -(\ Q, \neg P -(\ \}$.

The following definitions set technical details used in the remaining of the paper.

Definition 2.6. *Let γ be a candidate in τ, it is specified:*

1. *Conj(γ)*[8] $\equiv \{$*Conj(g)* $\mid g \in \gamma \}$.

[6] Again we use a finite collection of generalizations just for simplicity. All definitions can be adapted to infinite collections.

[7] In fact, given a default theory $\Delta = (W, D)$ whereon all defaults are prerequisite-free normal or semi-normal defaults, the same schema of translation can be used to obtain the corresponding axiomatic defeasible basis $\tau = (W, G)$.

[8] "Conj(γ)" is read "conjectures of γ".

2. *Rest*(γ)[9] $\equiv \{Rest(g) \mid g \in \gamma \}$.

We use the operator which gives the theorems of a theory Γ in classical logic, $Th(\Gamma)$, to define the theorems induced by a candidate γ in τ.

Definition 2.7. *Given a candidate* γ *in* τ, $Th_\tau(\gamma) \equiv Th(W \cup Conj(\gamma))$. $Th_\tau(\gamma)$ *is called the theory associated with* γ *in* τ.

Definition 2.8. *A candidate* γ *is consistent in* τ *iff* $Th_\tau(\gamma)$ *is consistent.*

In Example 2.1 the theory associated with the candidates are: $Th_\tau(\gamma_1) = Th(\{\text{Q}, \neg\text{P}\})$; $Th_\tau(\gamma_2) = Th(\{\text{P}, \text{Q}\})$; $Th_\tau(\gamma_3) = Th(\{\text{P}, \neg\text{P}\})$. In such case, γ_1 and γ_2 are consistent candidates while γ_3 is not.

Our proceeding to determine the theorems of a defeasible axiomatic basis is, then, to impose some conditions upon candidates to be considered an *expansion*. The theory associated with an expansion is what we call an *admissible set of theorems* of a defeasible axiomatic basis, this is our equivalent notion for *extension* in default logic.

If we regard the formulas in W and the generalizations in G, as premises of a defeasible axiomatic basis $\tau = (W, G)$, there is an important difference between monotonic and nonmonotonic logics. In monotonic logic, all premises are theorems of the theory, in nonmotononic logic, otherwise. Note that not all generalizations of a defeasible axiomatic basis can be in an expansion, for a generalization to be in an expansion, the expansion must be consistent and cannot prove exceptions of its own constituents.

Thus, finding expansions is a pre-processing phase yielding which generalizations will contribute to form an admissible set of theorems. Once this is done, theorems are determined by classical logic. In this manner, we separate the monotonic and nonmonotonic process for deriving theorems from a defeasible axiomatic basis.

In Example 2.1, $\gamma_2 = \{P{-}\!\!(\ Q, Q{-}\!\!(\ R\}$ is not an expansion because the exception of P–(Q is violated since $Th_\tau(\gamma_2) = \{P, Q\}$ proves the exception of P–(Q. $\gamma_3 = \{P{-}\!\!(\ Q, \neg P{-}\!\!(\ \}$ is not an expansion either, because $Th_\tau(\gamma_3) = \{P, \neg P\}$ is not consistent. On the other hand, $\gamma_1 = \{Q{-}\!\!(\ R, \neg P{-}\!\!(\ \}$ is a DLEF expansion as shown in Sect. 5

In order to present the properties which characterizes expansions we introduce the binary relations of rejection and exclusion on candidates.

Roughly, a candidate $\gamma_1$rejects a candidate γ_2 if the restrictions of γ_2 are proved in γ_1. Exclusion implies that the theories of the candidates are inconsistent. Rejection and exclusion involving generalizations are just specials cases when the candidate is a singleton.

Definition 2.9. *If* $\Gamma = \{P_1, .., P_n\}$ *is a collection of formulas in L:*

1. $\bigvee \Gamma \equiv \{P_1 \vee ... \vee P_n\}$;
2. $\neg\Gamma \equiv \{\neg P_1, ..., \neg P_n\}$.

[9] "Rest(γ)" is read "restrictions of γ".

Definition 2.10. *(Rejection and Exclusion)*

1. *γ rejects γ' in τ iff $Th_\tau(\gamma) \vdash \bigvee Rest(\gamma')$;*
2. *$\gamma_0 \subseteq \gamma'$ is a minimal part of γ' rejected by γ in τ iff γ rejects γ_0 in τ and for all $\gamma_1 \subsetneq \gamma_0$, γ does not reject γ_1 in τ.*
3. *γ excludes γ' in τ iff $\gamma \cup \gamma'$ is inconsistent in τ.*

In particular, we say that:

1. *g rejects γ* in τ, if $\{g\}$ rejects γ in τ, that is, $Th_\tau(\{g\}) \vdash \bigvee Rest(\gamma)$;
2. *γ rejects g* in τ, if γ rejects $\{g\}$ in τ, that is, $Th_\tau(\gamma) \vdash Rest(g)$.
3. *γ excludes g* in τ, if γ excludes $\{g\}$ in τ, that is, $\gamma \cup \{g\}$ is inconsistente.

In example 2.1, $\gamma_1 = \{Q{-}(\ R, \neg P{-}(\ \}$ rejects $\gamma_3 = \{P{-}(\ Q, \neg P{-}(\ \}$ for $Th_\tau(\gamma_1) = \mathrm{Th}(\{Q\ ,\ \neg\ P\}) \vdash \bigvee Rest(\gamma_3) = Q \vee \bot \equiv Q$. Moreover, $\{P{-}(\ Q\}$ is the minimal part of γ_3 rejected by γ_1.

Now, we are in a position to present the properties which will characterize expansions.

3 Properties

In this section we shall define some properties on candidates. Expansions are candidates satisfying some properties to be specified: soundness, completeness and exceptions-first. In fact, four properties because we define pointwise and global soundness. The flexibility of our approach allows us to define different logics just changing the properties imposed upon the expansions. This will be done in section 4. Our own logic `DLEF` is the only nonmonotonic logic - we claim that - to actually comply with the exceptions-first principle.

Soundness guarantees that a generalization does not belong to an expansion if its exception is proved in the associated theory. Global soundness, for instance, implies that, for finite candidates γ, the disjunction of the restrictions of the generalizations in γ is not proved in τ. Soundness, in any form, implies the consistency of the associated theory in τ.

Definition 3.1. *(Soundness)*

1. *A candidate γ is pointwise sound in τ iff for all $g \in \gamma$, $Th_\tau(\gamma) \nvdash Rest(g)$.*
2. *A candidate γ is globally sound in τ iff $Th_\tau(\gamma) \nvdash \bigvee Rest(\gamma)$.*

Example 3.1. Let $\tau = (\emptyset, \{P{-}(\ Q, R{-}(\ \neg Q\})$

There are three pointwise sound candidates, namely: $\gamma_1 = \{P{-}(\ Q\}$; $\gamma_2 = \{R{-}(\ \neg Q\}$; $\gamma_3 = \{P{-}(\ Q, R{-}(\ \neg Q\}$. Candidates γ_1 and γ_2 are also globally sound in τ. On the other hand, γ_3 is not globally sound because $Th_\tau(\gamma_3) \vdash \bigvee Rest(\gamma_3) = Q \vee \neg Q$.

The following lemmas can be easily proved.

Lemma 3.1. *Globally soundness implies pointwise soundness.*

Lemma 3.2. *If γ is pointwise sound in τ then γ is consistent in τ.*

Completeness states that a generalization is out of an expansion only if it is incompatible with the expansion, that is, either it makes the expansion inconsistent or its exception is proved in the expansion. In other words, completeness requires that as many generalizations as possible belong to an expansion.

Definition 3.2. *(Completeness) A candidate γ is complete in τ iff if $g \notin \gamma$ then γ rejects or excludes g in τ.*

Example 3.2. Let $\tau = (\emptyset, \{g_1 = P\text{–(}\ Q, g_2 = R\text{–(}\ \neg Q, g_3 = Q\text{–(}\ , g_4 = \neg Q\text{–(}\ \})$.
There are two complete candidates, namely: $\gamma_1 = \{P\text{–(}\ Q, \neg Q\text{–(}\ \}$; $\gamma_2 = \{R\text{–(}\ \neg Q, Q\text{–(}\ \}$.
Note that γ_1 is complete because g_2 is rejected and g_3 is excluded by γ_1. γ_2 is complete because g_1 is rejected and g_4 is excluded by γ_2.

The Exceptions-First Principle[10] captures a very important feature of reasoning with propositions subject to exceptions. Exceptions stipulate meta conditions to the applicability of a generalization. In order to apply a generalization, first it must be decided whether the exception is present. Only in case of its absence the generalization comes into play. By no means a proposition may intervene in the (non-)derivation of its own exception. This means that in case of a conflict between a generalization leading to an exception and the corresponding generalization, this guiding principle to defeasible reasoning asserts that the former must be derived. To illustrate the principle let us see a very simple defeasible axiomatic basis with only two generalizations (Ex. 3.3).

Example 3.3. Let $\tau = (\emptyset, \{g_1 = P\text{–(}\ Q,\ g_2 = Q\text{–(}\ \})$:

Notice that g_2 leads to an exception to g_1, hence these two generalizations are incompatible. There is no consensus in the AI community about the outcoming of this very simple defeasible axiomatic basis. Some argue that only g_2 should be derived and others argue for splitting the expansions: one applying g_1 and other applying g_2. Translating the example in Reiter's default logic (according with Def. 2.4), it applies only the second rule, and this has motivated some authors to "correct" default logic proposing variations which split this theory in two expansions. This is the case of the justified and constrained default logics [11,3] shown in section 5. We believe the EFP settles this issue out: only the second generalization should be applied.

The situation is worse than it might seem at first glance. In example 3.3 is not the case of an isolated artificial example. As a matter of fact, it is in the core of NMR. By essence, (we could say by definition) to reason is to chain partial conclusions until we reach a final conclusion. In NMR, this means to chain conclusions coming from defeasible propositions, which, in turn, might act as

[10] The exceptions-first principle was first enunciated in [18], its specification, however, was not made in properties like here, but, there, a partial ordering was introduced among the defeasible propositions of an axiomatic basis.

an exception to the next defeasible proposition. So, how the course of reasoning follows? The EFP is definite with no room for hesitation: derive the exception and preclude the corresponding defeasible proposition. None of the nonmonotonic logics that we know is aware of this guiding principle. This includes Reiter's default logic and its variants justified and constrained logics; McCarthy's circumscription [15]; Moore's Autoepistemic Logic [16]. In consequence, all of them engage in deriving the so called *anomalous extensions* [17].

Next, we define the Exceptions-First Principle.

Definition 3.3. *(Exceptions-First) A candidate γ complies with the exceptions-first principle in τ iff, for all candidates γ', if γ_0 is a minimal part of γ rejected by γ' then $\gamma - \gamma_0$ rejects or excludes γ' in τ.*

Example 3.4. Let $\tau = (\emptyset, \{P$–(Q, $(Q \rightarrow \neg P)$–(, Q–($\})$

Candidate $\gamma_1 = \{(Q \rightarrow \neg P)$–(, Q–($\}$ is globally sound and complete, as no consistent candidate rejects γ_1, it also satisfies EFP. On the other hand, candidate $\gamma_2 = \{P$–(Q, $(Q \rightarrow \neg P)$–($\}$ is globally (hence, also pointwise) sound and complete, however do not satisfy EFP, for $\gamma' = \{Q$–($\}$ rejects P–($Q \in \gamma_2$, but γ' is not rejected or excluded by $\{(Q \rightarrow \neg P)$–($\}$. We shall see in the sequel that γ_1 is a DLEF expansion and γ_2 is a default logic expansion. γ_2 is considered anomalous because the generalization P–(Q interfere to derive $\neg Q$, blocking the derivation of its exception in γ_2.

4 Defeasible Logic with Exceptions-First - DLEF

Our modular approach to determine the theorems from a defeasible axiomatic basis allows us to define an expansion simply listing the properties it must satisfy. An admissible set of theorems is simply the theory associated to the expansion. All nonmonotonic logics agree that an expansion must be consistent, do not violate exceptions (a generalization must not be a constituent of an expansion, if its exception is a theorem of this expansion) and it must be maximal, in the sense it contains as much generalizations as possible while preserving these properties. Apart from being consistent (there is no room for debate here, since the classical notion of consistency is adopted), there is more than one way to achieve the other requirements. These features give rise to different logics. We show some of these logics.

Here, we present the properties that define our own logic DLEF.

Definition 4.1. *A candidate γ is a DLEF expansion in τ iff*

1. *γ is globally sound in τ;*
2. *γ complies with the exceptions-first principle in τ;*
3. *γ is maximal with relation of both properties in τ, that is, if $\gamma \subsetneq \gamma'$ then γ' is not globally sound or do not comply with the exceptions-first principle in τ.*

γ being globally sound implies that it is consistent and do violate exceptions. Moreover, it means that it strongly commits to assumptions, a desirable property as argued by [3]. Further still, the second property is the EFP, and `DLEF` is the only nonmonotonic logic to comply with it.

Example 4.1 shows a representation of the emblematical bird problem in our formalism where A stands for animal, B for birds, P for penguins, F for flies, E for beaked animal and L for platypus.

Example 4.1. Let $\tau = (\{E, (E \rightarrow A)\}, \{(A \rightarrow \neg F)\text{–(}\ B, (B \rightarrow F)\text{–(}\ P, (E \rightarrow B)\text{–(}\ L\})$

This basis has only one `DLEF` expansion: $\gamma_1 = \{(B \rightarrow F)\text{–(}\ P, (E \rightarrow B)\text{–(}\ L\}$. Observe that $\gamma_2 = \{(A \rightarrow \neg F)\text{–(}\ B,\ (E \rightarrow B)\text{–(}\ L\}$ is pointwise sound and complete. However, γ_2 do not complies with EFP for γ_1 rejects $(A \rightarrow \neg F)\text{–(}\ B \in \gamma_2$, however, $\gamma_2 - \{(A \rightarrow \neg F)\text{–(}\ B\}$ do not reject or excludes γ_1. Not withstanding that, as we shall see in the next section, γ_2 corresponds to an extension in Reiter's default logic. In fact, to an anomalous extension. In γ_2 we conclude that the beaked animal is not a bird, simply from knowing that it is an animal. Clearly, this is not a supported line of reasoning.

A last word about the exceptions-first principle. The best mathematical analogy we can make is with is Zermelo and Fraenkel set theory [5]. A hierarchy on sets is postulated, a set can only have members from inferior levels. We claim that a similar hierarchy is presupposed in nonmonotonic reasoning: exceptions induce an inferential hierarchy on defeasible propositions [18]. A defeasible proposition, which participates in a derivation to an exception to another defeasible proposition, appears in an inferior level in the inferential hierarchy. Inference is, then, performed from the bottom up (the partial order on the hierarchy must be well founded). The EFP is thus warranted, the derivation of an exception to a defeasible proposition is performed in lower levels in relation to it, hence without its interference. In set theory a set cannot have as a member a set from the same or above level. In nonmonotonic reasoning a proposition cannot have as its exception, a fact from the same or above level. This is equivalent to say that a defeasible proposition cannot be *relevant* (meaning, it cannot participate in a derivation) to its own exception, lest the exception would be in a higher level.

This viewpoint provides us with a criterion on the good formation of defeasible axiomatic bases. To the induced binary relation to produce an hierarchy with a bottom level, it must be a strict well-founded order. Thus, a defeasible axiomatic basis is well-formed if the relation induced on the defeasible propositions by the exceptions is a strict well founded order (transitive and anti-reflexive). A study on well formed defeasible axiomatic bases is found in [14]. There it is proved that well-formed defeasible axiomatic bases always possess a `DLEF` expansion.

In next section we formulate definitions of expansions corresponding to Reiter's Default Logic, Justified Default Logic [11] and Constrained Default Logic [3]. In this manner, we will be capable to compare these default logics with `DLEF`.

5 Expansions for Default Logics

In this section we show that the diverse definitions of expansion in fact correspond to Reiter's original Default Logic and the Justified and Constrained variants. The ones acquainted with these works will notice how much simpler our equivalent formulation is. Unfortunately, for lack of space in this conference, it is out of the scope of this paper to present their definitions. For instance, while Lukaszewicz appeals to an awkward fixed-point of a binary operator on sets of formulas, justified expansion is defined just as a maximal sound set of generalizations (defaults). This also shows how flexible our modular approach is: just changing required properties different logics are obtained. The proofs of theorems are omitted - this can be considered challenge to the reader to try to prove them - but they appear in [23].

We name the expansions after the variants of default logic: justified, constrained or default expansion corresponding to, Justified Default Logic of Lukaszewicz [11]; Constrained Default Logic of Delgrande, Schaub and Jackson [3]; and the original Reiter's Default Logic [20], respectively.

5.1 Reiter's Default Logic - DL

This is the seminal approach all others are based into it. It has been introduced as a formal technique for handling reasoning with incomplete information. According to Reiter, an extension should be the smallest set of formulas containing the initial set of facts W, being deductively close and including each consequent of each applicable default. To formalize his notion of applicability, Reiter used a fixed-point operator while a default expansion is obtained, in our framework, simply requiring the stronger property of completeness instead of maximality. While the maximal variants (justified and constrained default logic) always have an expansion, default expansions may not exist for some axiomatic basis.

Definition 5.1. *A candidate γ is a default expansion in τ iff*

1. *γ is pointwise sound in τ;*
2. *γ is complete in τ.*

Theorems 5.1 and 5.2 shows the correspondence from default expansions and original definition of default logic.

Theorem 5.1. *If γ is a default expansion in $\tau = (W, G)$ then $E = Th_\tau(\gamma)$ is a default extension of $\Delta = (W, D)$, where D is the translation of G.*

Theorem 5.2. *If E is a default extension of $\Delta = (W, D)$ and GD^E_D is the set of generating defaults of E, then the translation of GD^E_D is a default expansion in $\tau = (W, G)$, where G is the translation of D.*

Theorem 5.3. *If γ is a* `DLEF` *expansion in $\tau = (W, G)$ then γ is a default expansion in $\tau = (W, G)$.*

Thus DLEF have less extension than Reiter's extension. This is for DLEF advantage, the "extra" extension are anomalous.

Default logic has been intensively studied and many proposal for modifying it have been advanced. Next, we show some of them.

5.2 Justified Default Logic - JDL

Justified default logic was introduced by Lukaszewicz [11] with the technical motivation of obtaining a local, iterative and constructive concept of extension. In Reiter's original formalism an extension cannot be construct applying the defaults successfully because a new default may dismiss an already applied default. Technically we say that default logic is not semi-monotonic.[11]

Lukaszewicz "fixes" this technical drawback once his logic is semi-monotonic. However, does he obtain the right inferences from a default theory? For instance, in the basic pattern of Example 3.3, JDL obtains two extensions. In term of inference this is just the opposite direction we take to justify the EFP. A justified expansion is defined as:

Definition 5.2. *A candidate γ is a justified expansion in τ iff*

1. *γ is pointwise sound in τ;*
2. *γ is maximal pointwise sound in τ, that is, if $\gamma \subsetneq \gamma'$ then γ' is not pointwise sound in τ.*

The next theorems states that indeed justified expansions correspond to justified extensions.

Theorem 5.4. *If γ is a justified expansion in $\tau = (W, G)$ then $E = Th_\tau(\gamma)$ is a justified extension wrt $J = \{Conj(\gamma) \cup \neg Rest(\gamma)\}$ of $\Delta = (W, D)$, where D is the translation of G.*

Theorem 5.5. *If E is a justified extension of $\Delta = (W, D)$ wrt J and $GD_D^{(E,J)}$ is the set of generating defaults of E, then the translation of $GD_D^{(E,J)}$ is a justified expansion in $\tau = (W, G)$, where G is the translation of D.*

5.3 Constrained Default Logic - CDL

Constrained Default Logic [3] has the property of "committing to assumptions", no inference is made based on contradictory assumptions (justifications of defaults). CDL incorporates a global notion of consistency to default logic. So, in CDL instead of checking consistency of defaults one by one separately, they are jointly considered. As a consequence, it is also semi-monotonic allowing a truly iterative definition of extension. Like JDL, inferentially it takes the opposite direction from EFP, and it also produces unwarranted extensions. A constrained expansion is defined as

[11] Reiter proves that the special (and limited) case of normal default theories are semi-monotonic.

Definition 5.3. *A candidate γ is a constrained expansion in τ iff*

1. *γ is globally sound in τ;*
2. *γ is maximal globally sound in τ, that is, if $\gamma \subsetneq \gamma'$ then γ' is not globally sound in τ.*

Theorems 5.6 e 5.7 show the correspondence from constrained expansions and original definition of CDL.

Theorem 5.6. *If γ is a constrained expansion in $\tau = (W, G)$ then $(E = Th_\tau(\gamma)$, $C{=}Th(W \cup Conj(\gamma) \cup \neg Rest(\gamma))$ is a constrained extension of $\Delta = (W, D)$, where D is the translation of G.*

Theorem 5.7. *If (E, C) is a constrained extension of $\Delta = (W, D)$ and $GD_D^{(E,C)}$ is the set of generating defaults of (E, C), then the translation of $GD_D^{(E,C)}$ is a constrained expansion in $\tau = (W, G)$, where G is the translation of D.*

Let see the expansions generated by the different versions in Example 5.1. The example is similar to one used in [3] to show weak and strong commitment to assumptions. For that matter, each variant behaves distinctly. JDL, weakly but not strongly commits to assumptions; CDL, weakly and strongly commits to assumptions; DL, neither weakly nor strongly commits to assumptions; and DLEF, strongly but not weakly commits to assumptions.

Example 5.1. $\tau = (\emptyset, \{P{-}(\ \neg Q, R{-}(\ Q, S{-}(\ P \vee R\})$:
JDL expansions: $\gamma_{J1} = \{P{-}(\ \neg Q,\ R{-}(\ Q\ \}$; $\gamma_{J2} = \{S{-}(\ P \vee R\ \}$.
CDL expansions: $\gamma_{C1} = \{P{-}(\ \neg Q\}$; $\gamma_{C2} = \{\ R{-}(\ Q\ \}$; $\gamma_{C3} = \{S{-}(\ P \vee R\ \}$.
DL expansion: $\gamma_{D1} = \{\ P{-}(\ \neg Q,\ R{-}(\ Q\}$.
DLEF expansions: $\gamma_{\mathtt{DLEF}1} = \{\ P{-}(\ \neg Q\ \}$; $\gamma_{\mathtt{DLEF}2} = \{R\ {-}(\ \ Q\ \}$.

The separation of $P{-}(\neg Q$ and $R{-}(\ Q$ in two different expansions sample that the logic strongly commits to assumptions. Moreover, the admission of an expansion with $S{-}(\ P \vee R$ comes of the fact of the logic weakly commits to assumptions.

6 Answer Set and Stratified Logic Programs

In this section, we analyse the relations between DLEF and Reiter's default logic with Answer Set Logic Programming. Stable models were introduced in [7] and since then it became the most successful approach for logic programming specially after 1999 when Marek and Truszczynski stablished the basis for the Answer Set Programming (ASP) paradigm [12]. It revitalizes the role of logic in many areas of AI, as, for instance, in planning and argumentation [24].

We show that Reiter's default logic and ASP produce the same models when default logic is restricted to the language of general logic programs. Marek and Truszczynski [13] proved a theorem to the same effect translating logic programs into default theories. We use the reverse strategy: we apply the concept of extensions to logic programs and show this is just another way of defining stable models.

Definite Logic programs are set of clauses of the form: $A \leftarrow A_1, \ldots, A_n$, where, A, A_i are all atoms[12]. An *interpretation* for a logic program is a set of atoms. Associated to a definite logic program, Π, there is an operator on interpretations of Π defined as follows: $F_\Pi(I) = \{A | A \leftarrow A_1, \ldots, A_n \in \Pi \text{ and } A_1, \ldots, A_n \in I\}$.

The operator F_Π, being monotonic, has a least fixed point denoted $\mathbf{lfp}(F_\Pi)$. This is called the *minimal model* of the program Π [22]. $\mathbf{lfp}(F_\Pi)$ can also be characterized as the least closed set of F_Π [1].

General logic programs are set of clauses including negation, that is, clauses of the form: $A \leftarrow A_1, \ldots, A_n, \text{ not } B_1, \ldots, \text{ not } B_k$ (A, A_i, B_i are atoms, as before). The ASP semantics associates an operator F_Π on interpretations for a general program Π. For any interpretation I let Π_I be the program obtained from Π by deleting:

1. Each clause that has a literal not B in its body with $B \in I$;
2. All negative literals in the remaining clauses.

Now, Π_I is a definite program, define $F_\Pi(I) = \mathbf{lfp}(F_{\Pi_I})$. F_Π is not monotonic and therefore it might have one, none or several fixed points. The *stable model for a general program* Π is, then, defined as a minimal fixed point of F_Π, when there is at least one.

In turn, Reiter's default logic can be adapted for logic programs. Given a general logic program Π, define Γ_Π on interpretations as follows: $\Gamma_\Pi(I)$ is a minimal set of atoms such that:

(i) for all $A \leftarrow A_1, \ldots, A_n, \text{ not } B_1, \ldots, \text{ not } B_k \in \Pi$, if $A_1, \ldots, A_n \in \Gamma_\Pi(I)$ and $B_1, \ldots, B_k \notin I$, then $A \in \Gamma_\Pi(I)$. We say that I is an extension for Π iff $I = \Gamma_\Pi(I)$.

This "Reiter's semantics" for logic programs coincide with ASP.

Theorem 6.1. *Let Π be a general logic program, I is a stable model for Π iff I is an extension for Π.*

Proof. Just observe that for all interpretations I of Π, $\Gamma_\Pi(I) = F_\Pi(I)$. To show that $\Gamma_\Pi(I) \subseteq F_\Pi(I)$ it is enough to show that $F_\Pi(I)$ satisfies condition (i) in the definition of $\Gamma_\Pi(I)$. Suppose $A \leftarrow A_1, \ldots, A_n, \text{ not } B_1, \ldots, \text{ not } B_k \in \Pi$. Assume $A_1, \ldots, A_n \in F_\Pi(I)$ and $B_1, \ldots, B_k \notin I$.Hence, $A \leftarrow A_1, \ldots, A_n \in \Pi_I$. As, $A_1, \ldots, A_n \in F_\Pi(I)$ then, A $\in F_{\Pi_I}(F_\Pi(I))$, for $F_\Pi(I)$ is the least fixed point of F_{Π_I} . For the converse, it is enough to show that $F_{\Pi_I}(\Gamma_\Pi(I)) \subseteq \Gamma_\Pi(I)$,for $F_\Pi(I)$ is the least fixed point of F_{Π_I}. Let $A \leftarrow A_1, \ldots, A_n \in \Pi_I$ and $A_1, \ldots, A_n \in \Gamma_\Pi(I)$.There are two cases:

1. $A \leftarrow A_1, \ldots, A_n \in \Pi$, then, by the condition (i) in the definition of $\Gamma_\Pi(I)$, $A \in \Gamma_\Pi(I)$.

[12] Again, for simplicity's sake we adopt a propositional language.

2. $A \leftarrow A_1, \ldots, A_n,$ not $B_1, \ldots,$ not $B_k \in \Pi$, whereon, by the construct of Π_I, $B_1, \ldots, B_k \notin I$.Hence, by the condition (i) in the definition of $\Gamma_\Pi(I)$, A $\in \Gamma_\Pi(I)$.

Hence, ASP produce just the same models as Reiter's default semantics for logic programs. Therefore, how can one, in one hand, acknowledge that Reiter's default logic produces anomalous extensions, and, on the other, claim that ASP solves the anomalous extension problem? In fact, the "solution" is not in ASP, but in the restricted language, which does not allow the expression of the anomalous extensions. Indeed, Lifschitz argues along this line in [10]. He states in section 2.4, pg. 43, that "Paradoxically, limitations of the language of logic programs play a positive role in this case (the "Yale Shooting Problem") by eliminating some of the "bad" representational choices that are available when properties of actions are described in default logic". The EFP is a better explanation for the correct representational choice.

On the other hand `DLEF` semantics for logic programs with negation can also be defined, then it can be shown that `DLEF`, Reiter's and ASP semantics all coincide for stratified general programs [19]. The "Yale Shooting Problem", for instance, is represented by a stratified logic program. Compliance with exceptions-first principle in stratified programs is the effective reason why ASP semantics solves the anomalous extensions problem.

In `DLEF`, as we mention at the end of section 4, a syntactic order on the defeasible propositions of a defeasible axiomatic basis can be defined [18]. When this order is strict (transitive and anti-reflexive) we say that the defeasible axiomatic basis is well-formed. We have shown in [14] that well-formed basis always have at least an extension. There is a direct relation between this order and stratified logic programs. Translating general clauses in logic programs to generalizations in defeasible axiomatic basis in the natural way, it can be shown that stratified logic programs are translated inyo well-formed defeasible axiomatic basis [19]. Moreover, the deductive closure of a stable model of the logic program is a `DLEF` extension of the respective defeasible axiomatic basis. This is another way of explaining why stratified logic program have a stable model.

7 Future Work: Inductive Definitions

As future work, we are investigating how to treat logics as formal systems or inductive definitions [1]. Monotonic logics are related to monotone inductive definitions and then to monotone operators. Nonmonotonic logics correspond to nonmonotonic inductive definitions and operators. Monotone operators have a constructive least fixed point, and this is defined as the set inductively generated. Nonmonotone operators may have one, none or several fixed points, and there is no constructive way to determine them. It is usual to define, then, the set generated by a nonmonotone inductive definition as a fixed point of the respective nonmonotone operator, when it exists (what other option could be?). This is the case of Reiter's default logic and ASP, for instance.

At the moment, we are developing the following line of reasoning. Nonmonotone inductive definitions do not have fixed points because they are not *predicative* as monotone inductive definitions are. Predicativity was very polemic at the beginning of the XX century and it is in the core of the Russell's paradox in set theory, for instance, the set $\{x|x \notin x\}$ is not predicative [6]. We reckon the exceptions-first principle as a way of restoring predicativity in nonmonotone inductive definitions. Well-formed inductive definitions always have a fixed point in compliance with the exceptions-first principle. It might have others not in compliance with EFP, these will be the anomalous fixed-points. For instance, stratified logic programs (a logical program can be regarded as an inductive definition, see [4]) are well-formed in the sense of the EFP. That is why they have a fixed point or stable model. The EFP states three important things:

1. Not all inductive definitions are well-formed.
2. Not all fixed points of a well-formed nonmonotone inductive definition should be regarded as an inductively generated set, only the ones that are predicative.
3. EFP and predicativity - not the restriction on the expressivity of language - solve and explain the anomaly of some fixed points of nonmonotone inductive definitions.

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